Global phase time and path integral
for string cosmological models

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A global phase time is identified for homogeneous and isotropic cosmological models yielding from the low energy effective action of closed bosonic string theory. When the Hamiltonian constraint allows for the existence of an intrinsic time, the quantum transition amplitude is obtained by means of the usual path integral procedure for gauge systems.

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1. Introduction

An essential property of gravitational dynamics is that the canonical Hamiltonian vanishes on the physical trajectories of the system; the constraint $\mathcal{H} \approx 0$ reflects that the evolution is given in terms of a parameter τ which does not have physical significance. This leads to a fundamental difference between ordinary quantum mechanics and the quantization of gravitation, because the existence of a unitary quantum theory is related to the possibility of defining the time as an absolute parameter. The identification of a global phase time [1] can therefore be considered as the previous step before quantization [2].

In the theory of gravitation the Hamiltonian not only generates the dynamical evolution, but it also acts as a generator of gauge transformations which connect any pair of succesive points on each classical trajectory of the system. While the dynamics is given by a spacelike hypersurface evolving in spacetime, including arbitrary local deformations which yield a multiplicity of times, the same motion can be generated by gauge transformations [3]. It is therefore natural to think that the gauge fixing procedure can be a way to identify a global time.

However, as the action of gravitation is not gauge invariant at the boundaries, this idea could not be used, in principle, to give a direct procedure for deparametrizing minisuperspaces: while ordinary gauge systems admit canonical gauges $\chi(q^i, p_i, \tau) = 0$, only derivative gauges would be admissible for cosmological models [4,5]. These gauges cannot define a time in terms of the canonical variables. At the quantum level this has the consequence that the usual path integral for ordinary gauge systems could not be applied.

In the present paper we give a proposal for solving these problems in the case of isotropic and homogeneous cosmological models resulting from the bosonic closed string theory. We define a canonical transformation so that the action of the minisuperspaces is turned into that of an ordinary gauge system [6]; then we use canonical gauge conditions to identify a global phase time in terms of the canonical variables for most possible values of the parameters characterizing the models. When the Hamiltonian has a potential with

2

a definite sign an intrinsic time $t(q)$ is defined, and the quantum transition amplitude for separable models is obtained in the form of a path integral for an ordinary gauge system; the $\tau-$dependent gauge choice used to identify the time determines the time integration parameter and the observables to be fixed at the end points. Differing from our previous analysis [6] (which was restricted to simple models within the framework of general relativity), now we also obtain an extrinsic time $t(q,p)$ for the models, and, more important, we give a reduction procedure which leads to a conserved true Hamiltonian, thus making more clear the meaning of the quantization.

2. String cosmology models

2.1. *Gauge invariant action*

The cosmological field equations yielding from the low energy action of string theory show a remarkable T–duality symmetry that appears manifestly in terms of redefinition of the fields. The duality properties of the models make string cosmology very interesting, since it makes possible to propose a pre–big bang phase for the universe [7]. The quantization of string cosmological models has been analyzed in the context of the graceful exit problem (for a detailed discussion see references [8] and [9], and references therein), and it has been remarked [10,11] that a careful discussion of the subtleties that are typical of the quantization of gauge systems is required. In the present work the formal aspects of the problem are studied, and we give a solution for the models whose Hamilton–Jacobi equation is separable; we show that some results are valid also for more general models.

The massless states of bosonic closed string theory are the dilaton ϕ, the two-form field $B_{\mu\nu}$ and the graviton $g_{\mu\nu}$ which fixes the background geometry. The low energy effective action that describes the long-wavelength limit of the massless fields dynamics is (written in Einstein frame)

$$S = \frac{1}{16\pi G_D} \int d^D x \sqrt{-g} \left(R - c e^{2\phi/(D-2)} - \frac{1}{(D-2)}(\partial\phi)^2 - \frac{1}{12} e^{-4\phi/(D-2)}(dB)^2 \right) \quad (1)$$

3

where $c = \frac{2}{3\alpha'}(D - 26)$, being α' the Regge slope, and dB is the exterior derivative of the field $B_{\mu\nu}$. In this paper we consider c as an arbitrary real parameter.

The Euler-Lagrange equations yielding from the action (1) admit homogeneous and isotropic solutions in four dimensions [12,13,14,15]. Such solutions present a Friedmann–Robertson–Walker form for the metric, namely

$$ds^2 = N^2(\tau)d\tau^2 - e^{2\Omega(\tau)}\left(\frac{dr^2}{1 - kr^2} + r^2 d\theta^2 + r^2 \sin^2\theta d\varphi^2\right). \tag{2}$$

For the dilaton ϕ and the field strength $\mathbf{H} = dB$, the homogeneity and isotropy constraints demand

$$\mathbf{H}_{ijk} = \lambda \varepsilon_{ijk} \qquad \phi = \phi(\tau), \tag{3}$$

where ε_{ijk} is the volume form on the constant-time surfaces and λ is a real number. The Einstein frame action for this system in four dimensions is given by

$$S = \frac{1}{2}\int d^4x\sqrt{-g}Ne^{3\Omega}\left[-\frac{\dot{\Omega}^2}{N^2} + \frac{\dot{\phi}^2}{N^2} - 2ce^{\phi} + \delta_{\lambda,0}ke^{-2\Omega} - \delta_{k,0}\lambda^2 e^{-6\Omega-2\phi}\right], \tag{4}$$

where the $\delta's$ are introduced to consider the cases of a flat model with two-form field different from zero, and a closed or open model with $\mathbf{H}_{\mu\nu\rho} = 0$. If we put the action in the Hamiltonian form

$$S = \int d\tau\left[\pi_{\Omega}\dot{\Omega} + \pi_{\phi}\dot{\phi} - N\mathcal{H}\right] \tag{5}$$

the canonical Hamiltonian is

$$\mathcal{H} = \frac{1}{2}e^{-3\Omega}\left(-\pi_{\Omega}^2 + \pi_{\phi}^2 + 2ce^{6\Omega+\phi} - \delta_{\lambda,0}ke^{4\Omega} + \delta_{k,0}\lambda^2 e^{-2\phi}\right) \approx 0. \tag{6}$$

As the constraint is quadratic in the momenta, the action is not gauge invariant at the boundaries of the trajectories [4,5,6]; however, if the Hamilton-Jacobi equation associated with $\mathcal{H}$ is separable, the action can be turned into that of an ordinary gauge system by improving it with gauge invariance at the end points [6]. We shall begin by considering the following generic form for the scaled Hamiltonian $H \equiv 2e^{3\Omega}\mathcal{H}$:

$$H = -\pi_{\Omega}^2 + \pi_{\phi}^2 + 4Ae^{n\Omega+m\phi} \approx 0, \tag{7}$$

where A is an arbitrary real constant and $m \neq n$. In general, this Hamiltonian is not separable in terms of the original canonical variables. Then we define

$$x \equiv \left(\frac{2}{n+m}\right) e^{(n+m)(\Omega+\phi)/2}, \qquad y \equiv \left(\frac{2}{n-m}\right) e^{(n-m)(\Omega-\phi)/2} \tag{8}$$

so that dividing H by $(n^2 - m^2)xy > 0$ we can define the equivalent constraint

$$H' \equiv \frac{H}{(n^2 - m^2)xy} = -\pi_x\pi_y + A \approx 0. \tag{9}$$

The solution of the corresponding Hamilton–Jacobi equation $-(\partial W/\partial x)(\partial W/\partial y) + A = E'$ which is obtained by matching the integration constants α, E' to the momenta $\overline{P}, \overline{P}_0$ is

$$W(x, y, \overline{P}_0, \overline{P}) = \overline{P}x + y\left(\frac{A - \overline{P}_0}{\overline{P}}\right).$$

The solution W generates a canonical transformation $(q^i, p_i) \to (\overline{Q}^0, \overline{Q}, \overline{P}_0, \overline{P})$ which identifies H' with $\overline{P}_0$. The variables $(\overline{Q}, \overline{P})$ are conserved observables because $[\overline{Q}, H'] = [\overline{P}, H'] = 0$, so that they would not be appropriate to characterize the dynamical evolution. The function

$$F = P_0\overline{Q}^0 + f(\overline{Q}, P, \tau) \tag{10}$$

generates a second transformation in the space of observables $(\overline{Q}, \overline{P}) \to (Q, P)$, such that the new Hamiltonian $K = NP_0 + \partial f/\partial\tau$ does not vanish, and Q is a non conserved observable because $[Q, H'] = [P, H'] = 0$ but $[Q, K] \neq 0$. For Q^0 we have $[Q^0, H'] = [Q^0, P_0] = 1$, and then Q^0 can be used to fix the gauge [16]. If we choose

$$f = \overline{Q}P + T(\tau)/P \tag{11}$$

with $T(\tau)$ a monotonic function then the new canonical variables are given by

$$P_0 = -\pi_x\pi_y + A, \qquad P = \pi_x,$$

$$Q^0 = -\frac{y}{P}, \qquad Q = x - \left(\frac{y(A - P_0) + T(\tau)}{P^2}\right) \tag{12}$$

($P = \pi_x$ cannot be zero on the constraint surface). The coordinates and momenta (Q^i, P_i) describe an ordinary gauge system with a constraint $P_0 = 0$ and a true Hamiltonian $\partial f/\partial\tau = (1/P)(dT/d\tau)$ which conmutes with K. Its action is

$$S[Q^i, P_i, N] = \int_{\tau_1}^{\tau_2} \left(P\frac{dQ}{d\tau} + P_0\frac{dQ^0}{d\tau} - NP_0 - \frac{1}{P}\frac{dT}{d\tau} \right) d\tau. \tag{13}$$

If we write $\mathcal{S}$ in terms of the original variables we must add end point terms [17,6] of the form [6] $B = \overline{Q}^i\overline{P}_i - W + QP - f$ so that

$$S[\Omega, \phi, \pi_\Omega, \pi_\phi, N] = \int_{\tau_1}^{\tau_2} \left(\pi_\phi\frac{d\phi}{d\tau} + \pi_\Omega\frac{d\Omega}{d\tau} - N\mathcal{H} \right) d\tau + B(\tau_2) - B(\tau_1), \tag{14}$$

where

$$B(\tau) = \frac{1}{(\pi_\Omega + \pi_\phi)} \left[\frac{\pi_\phi^2 - \pi_\Omega^2 + 4Ae^{n\Omega+m\phi}}{n - m} + 4Ae^{(n+m)(\Omega+\phi)/2} \left(\frac{2e^{(n-m)(\Omega-\phi)/2}}{n - m} + \frac{T(\tau)}{A} \right) \right].$$

As $\pi_x = \overline{P} = (1/2)(\pi_\Omega + \pi_\phi)e^{-(n+m)(\Omega+\phi)/2}$ we can write

$$B(\tau) = -Q^0P_0 - 2A\left(Q^0 - \frac{T(\tau)}{A\overline{P}} \right).$$

Under a gauge transformation generated by $\mathcal{H}$ we have $\delta_\epsilon B = -\delta_\epsilon S$, so that the action $\mathcal{S}$ is effectively endowed with gauge invariance over the whole trajectory and canonical gauge conditions are admissible.

2.2. *Extrinsic time*

A global phase time t must verify $[t, \mathcal{H}] > 0$ [1], but as $\mathcal{H} = \mathcal{F}(\Omega, \phi)H' = \mathcal{F}(\Omega, \phi)P_0$ with $\mathcal{F} > 0$, then if t is a global phase time we also have $[t, P_0] > 0$. Because $[Q^0, P_0] = 1$, an extrinsic time can be identified by imposing a $\tau-$dependent gauge of the form $\chi \equiv Q^0 - T(\tau) = 0$ and defining $t \equiv T$. We then obtain

$$t(\Omega, \phi, \pi_\Omega, \pi_\phi) = Q^0 = \frac{4e^{n\Omega+m\phi}}{(m - n)(\pi_\Omega + \pi_\phi)}. \tag{15}$$

Using the constraint equation (7) we can write $t(\pi_\Omega, \pi_\phi) = (n - m)^{-1}(\pi_\phi - \pi_\Omega)/A$. For the scaled constraint $H = 2e^{3\Omega}\mathcal{H}$ with $k = \lambda = 0$ we have $4A = 2c$, $n = 6$, $m = 1$. Then the extrinsic time is

$$t(\Omega, \phi, \pi_\Omega, \pi_\phi) = -\frac{4e^{6\Omega+\phi}}{5(\pi_\Omega + \pi_\phi)}. \tag{16}$$

We can go back to the constraint H with $k \neq 0$ and evaluate $[t, H]$. For an open model $(k = -1)$ a simple calculation gives that $[t, H] > 0$ for both $c < 0$ and $c > 0$. For the case $k = 1$, instead, an extrinsic global phase time is $t(\pi_\Omega, \pi_\phi) = (2/5c)(\pi_\phi - \pi_\Omega)$ if $c < 0$.

In the case of the scaled constraint with $c = k = 0$ we have $4A = \lambda^2$, $n = 0$, $m = -2$, and the extrinsic time reads

$$t(\Omega, \phi, \pi_\Omega, \pi_\phi) = -\frac{2e^{-2\phi}}{\pi_\Omega + \pi_\phi}. \tag{17}$$

If we then consider $c \neq 0$ and we compute the bracket $[t, H]$ we find that this is positive definite if $c < 0$. Hence the time given by (17) is a global phase time for this case.

2.3. *Intrinsic time and path integral*

The action (13) can be used to compute the amplitude for the transition $|Q_1, \tau_1 > \rightarrow |Q_2, \tau_2 > (Q^0$ is a spurious degree of freedom for the gauge system) by means of a path integral in the form

$$< Q_2, \tau_2 | Q_1, \tau_1 > = \int DQ^i DP_i DN \delta(\chi) \, |[\chi, P_0]| \, \exp\left[i \int_{\tau_1}^{\tau_2} \left(P_i \frac{dQ^i}{d\tau} - NP_0 - \frac{\partial f}{\partial \tau}\right) d\tau\right]. \tag{18}$$

Here $|[\chi, P_0]|$ is the Fadeev-Popov determinant; because the constraint is simply $P_0 = 0$, canonical gauges are admissible. But what we want to obtain is the amplitude $< q_2^i | q_1^i >$, so that we should show that both amplitudes are equivalent. This is fulfilled if the paths are weighted in the same way by $\mathcal{S}$ and S and if Q and τ define a point in the original configuration space, that is, if a state $|Q, \tau >$ is equivalent to $|q^i >$. This is true only if there exists a gauge such that $\tau = \tau(q^i)$, and such that on the constraint surface the boundary terms in (14) vanish [6].

The existence of a gauge condition yielding $\tau = \tau(q^i)$ is closely related to the existence of an intrinsic time [18]. A (globally good) gauge such that $\tau = \tau(q^i)$ should be given by a function $\chi(q^i, \tau) = 0$ fulfilling $[\chi, \mathcal{H}] \neq 0$, while a function $t(q^i, p_i)$ is a global phase time if $[t, \mathcal{H}] > 0$. Because the supermetric G^{ik} does not depend on the momenta, a function

7

$t(q)$ is a global phase time if the bracket

$$[t(q), \mathcal{H}] = [t(q), G^{ik} p_i p_k] = 2 \frac{\partial t}{\partial q^i} G^{ik} p_k$$

is positive definite. For a constraint whose potential can be zero for finite values of the coordinates, the momenta p_k can be all equal to zero at a given point, and $[t(q), \mathcal{H}]$ can vanish. Hence an intrinsic time can be identified only if the potential has a definite sign.

On the constraint surface $H' = P_0 = 0$ the terms $B(\tau)$ clearly vanish in the canonical gauge

$$\chi \equiv Q^0 - \frac{T(\tau)}{A\overline{P}} = 0 \tag{19}$$

which is equivalent to $T(\tau) = 2(m - n)^{-1} A e^{(n-m)(\Omega - \phi)/2}$, and then it defines $\tau = \tau(\Omega, \phi)$. As $P = \overline{P}$ and thus $Q^0 \overline{P} = -y(\Omega, \phi)$, an intrinsic time t can be defined as

$$t \equiv \frac{\eta T}{2A}$$

if we apropriately choose η. We have $[t, H'] = (\eta/2)[Q^0 \overline{P}, P_0] = (\eta/2)\overline{P}$, and because $\overline{P} = \pi_x$ then to ensure that t is a global phase time we must choose $\eta = sign(\pi_x) = sign(\pi_\Omega + \pi_\phi)$.

In the case $A > 0$ it is $|\pi_\Omega| > |\pi_\phi|$ (so that $sign(\pi_x) = sign(\pi_\Omega)$) and the constraint surface splits into two disjoint sheets identified by the sign of π_Ω; in the case $A < 0$ it is $|\pi_\phi| > |\pi_\Omega|$ and the two sheets of the constraint surface are given by the sign of π_ϕ. Hence in both cases η is determined by the sheet of the constraint surface on which the system evolves; we therefore have that for $A > 0$ the intrinsic time can be written as

$$t(\Omega, \phi) = \left(\frac{1}{m - n}\right) sign(\pi_\Omega) e^{(n-m)(\Omega - \phi)/2}, \tag{20}$$

while for $A < 0$ we have

$$t(\Omega, \phi) = \left(\frac{1}{m - n}\right) sign(\pi_\phi) e^{(n-m)(\Omega - \phi)/2}. \tag{21}$$

For the constraint with $k = \lambda = 0$ the intrinsic time is

$$t(\Omega, \phi) = -\frac{1}{5} sign(\pi_\Omega) e^{5(\Omega - \phi)/2} \qquad \text{if} \qquad c > 0,$$

and

$$t(\Omega, \phi) = -\frac{1}{5} sign(\pi_\phi) e^{5(\Omega-\phi)/2} \qquad \text{if} \qquad c < 0.$$

By evaluating the bracket $[t, H]$ for H with $k \neq 0$ we find that the intrinsic time obtained in the case $c > 0$ is also a time for an open model ($k = -1$), and the time for $c < 0$ is a time also for $k = 1$.

In the case of the constraint with $c = k = 0$ we obtain

$$t(\Omega, \phi) = -\frac{1}{2} sign(\pi_\Omega) e^{(\Omega-\phi)},$$

and a simple calculation shows that this is also a global phase time for a more general model with $c > 0$.

Because we have shown that there is a gauge such that $\tau = \tau(q^i)$ and which makes the endpoint terms vanish, we can obtain the amplitude for the transition $|\Omega_1, \phi_1 > \rightarrow |\Omega_2, \phi_2 >$ by means of a path integral in the variables (Q^i, P_i) with the action (13). This integral is gauge invariant, so that we can compute it in any canonical gauge. According to (18), on the constraint surface $P_0 = 0$ and with the gauge choice (19), the transition amplitude is

$$< \phi_2, \Omega_2 | \phi_1, \Omega_1 > = \int DQ DP \exp\left[i \int_{T_1}^{T_2} \left(P dQ - \frac{1}{P} dT \right) \right], \qquad (22)$$

where the end points are given by

$$T_a = \left(\frac{2A}{m-n} \right) e^{(n-m)(\Omega_a - \phi_a)/2}$$

($a = 1, 2$); because on the constraint surface and in gauge (19) the true degree of freedom reduces to $Q = x$, then the boundaries of the paths in phase space are

$$Q_a = \left(\frac{2}{n+m} \right) e^{(n+m)(\Omega_a + \phi_a)/2}.$$

For the Hamiltonian with $\lambda = 0$ and null curvature the end points are given by $T_a = -(c/5) e^{5(\Omega_a - \phi_a)/2}$, while $Q_a = (2/7) e^{7(\Omega_a + \phi_a)/2}$. In the case of $c = k = 0$ we have $T_a = -(\lambda^2/4) e^{(\Omega_a - \phi_a)}$ and $Q_a = -e^{-(\Omega_a + \phi_a)}$.

9

After the gauge fixation we have obtained the path integral for a system with one physical degree of freedom. A point to be remarked is that in our previous work [6] the reduction procedure yielded a true Hamiltonian which was analogous to that of a massless particle with a time dependent potential; such a potential leads to particle creation, so that the meaning of the minisuperspace quantization would not be completely clear. Here, instead, we have avoided this difficulty because we have obtained a true Hamiltonian $1/P$ which does not depend on time (see eq. (22)).

3. Discussion

We have been able to use canonical gauge conditions for deparametrizing homogeneous and isotropic cosmological models coming from the low energy dynamics of bosonic closed string theory and, simultaneously, to obtain the quantum transition amplitude in a simple form which clearly shows the separation between true degrees of freedom and time.

We have analized models of two types: 1) models with homogeneous dilaton field and vanishing antisymmetric $B_{\mu\nu}$ field ($\lambda = 0$); 2) models representing flat universes ($k = 0$) with homogeneous dilaton and non vanishing antisymmetric field. For the cases considered we have been able to identify a global phase time. In the cases $\lambda = 0,\ k = 0,\ c \neq 0$ and $\lambda \neq 0,\ k = 0,\ c = 0$ the Hamiltonian is easily separable and the potential has a definite sign. Thus, an intrinsic time can be found and the quantum transition amplitude is obtained by means of a path integral in the new variables (Q^i, P_i) describing an ordinary gauge system. The canonical gauge used to define the time determines the integration parameter and the variables to be fixed at the boundaries.

Once we have found a time t for the inmediately separable models, we have identified the extended region of the parameter space where t is a global phase time. In fact, a simple prescription can be given to determine whether an extrinsic time for a system described by a given Hamiltonian is also a time for a system described by a more general constraint. We have defined $H' = g^{-1}(q)H$ with $g > 0$, and because we matched $P_0 \equiv H'$,

10

then $t \equiv Q^0$ fulfills $[t, H'] = 1$ (and then $[t, H] = g > 0$ on the surface $H = 0$). If we consider an extended constraint $\tilde{H} = g(q)H' + h$ and we calculate the bracket of t with $\tilde{H}$ we obtain

$$[t, \tilde{H}] = g + H'[t, g] + [t, h].$$

Using that $\tilde{H} \approx 0$ we have that the condition

$$[t, \tilde{H}] = g - g^{-1}h[t, g] + [t, h] > 0$$

must hold on the (new) constraint surface if t is a time for the system described by $\tilde{H}$. For the system associated to the constraint (7), from (8) and (9) we have that $g = 4e^{n\Omega + m\phi}$; if we add a term of the form $h = \alpha e^{r\Omega + s\phi}$ to H the condition turns to be

$$\alpha \frac{e^{r\Omega + s\phi}}{(\pi_\phi + \pi_\Omega)^2} \left[\frac{(n + m) - (r + s)}{n - m} \right] > -1.$$

We have restricted our analysis to the formal aspects of minisuperspace quantization. A complete discussion about the limits of such approximation as well as an analysis of the application of our method to the graceful exit problem would require a detailed knowledge of the effective potential for the dilaton. If the effective potential leads to a separable Hamilton–Jacobi equation the application of our procedure would result straightforward. The minisuperspaces that we have quantized admit an intrinsic time; however, an intrinsic time can be defined only if the constraint surface splits into two disjoint sheets. If the complete Hamiltonian including the effective potential is separable but admits only an extrinsic time, the variables to be fixed at the boundaries in the path integral should involve not only the original coordinates but also the momenta; this point would require a further discussion.

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